BOOK
A Divine Language: Learning Algebra, Geometry, and Calculus at the Edge of Old Age

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SYNOPSIS [From the publisher]
"Decades after struggling to understand math as a boy, Alec Wilkinson decides to embark on a journey to learn it as a middle-aged man. What begins as a personal challenge—and it's challenging—soon transforms into something greater than a belabored effort to learn math. Despite his incompetence, Wilkinson encounters a universe of unexpected mysteries in his pursuit of mathematical knowledge and quickly becomes fascinated; soon, his exercise in personal growth (and torture) morphs into an intellectually expansive exploration."

"I felt a kinship with Carl Jung, who described math class as "sheer terror and torture," since he was 'amathematikos,' which means something like nonmathematical."

"I am by nature a self-improver. I have read Gibbon, I have read Proust. I read the Old and New Testaments and most of Shakespeare. I studied French. I have meditated. I jogged. I learned to draw, using the right side of my brain. A few years ago, I decided to see if I could learn simple math."

"A letter that the French philosopher Simone Weil wrote to a pupil in 1934. One ought to try to learn complicated things by finding their relations in "commonest knowledge," Weil writes. "It is for this reason that you ought to study, and mathematics above all."

"I was sixty-five when I started. Also, I wanted especially to study calculus because I never had. I didn't even know what it was—I quit math after feeling that with Algebra II I had pressed my luck as far as I dared."

"After a time my studies began to occupy two channels. One channel involved trying to learn algebra, geometry, and calculus, and the other channel involved the things they introduced me to and led me to think about."

"What did I learn? Among other things, that while mathematics is the most explicit artifact that civilization has produced, it has also provoked many speculations that do not appear capable of being settled."

"I had it in for mathematics, for what I recalled of its self-satisfaction, its smugness, and its imperiousness. It had abused me, and I felt aggrieved. I was returning, with a half century's wisdom, to knock the smile off math's face."

"In Ulysses James Joyce writes that the present is the drain that the future goes down on its way to becoming the past."

"The algebra that the ancients knew in Egypt, India, Babylonia, Greece, and Persia is essentially the algebra that is taught in high school."

"Mathematics is severe and faultless. It became the language of science because of its precision. The theory of relativity can be written in prose, but $e = mc^2$ is more succinct."

"I wish someone had said on my first day in algebra class, "To start, all you need to know is that you are answering a problem whose solution, instead of involving a single unknown, as it does in arithmetic, involves a second unknown, which we call x."

"It requires unusual abilities to become a mathematician, that and years of painful training in which the intellect is forced to bend upon itself." —David Berlinski, A Tour of the Calculus

"There is a belief among certain academics that a subject is less efficiently learned from an adept than from someone who is studying it or has just finished studying it. The adept's long acquaintance makes it difficult for him or her to see the subject in its simpler terms or to appreciate what it is like to approach the subject as a greenhorn."

"In *An Introduction to Mathematics,* by Alfred North Whitehead, I come across simple information that I might have found helpful. "The ideas of any and of some are introduced into algebra by the use of letters, instead of the definite numbers of arithmetic. Thus, instead of saying that 2 + 3 = 3 + 2, in algebra we generalize and say that, if x and y stand for any two numbers, then x + y = y + x."

"Possibly not everyone knows that algebra is thought to be the contribution, although maybe not entirely the invention, of a Persian mathematician and librarian named Muhammad ibn Musa al-Khwarizmi, who lived in Baghdad in the ninth century."

"Ramus is the source for the assertion that "there was a certain learned mathematician who sent his algebra, written in the Syriac language, to Alexander the Great, and he named it almucabala, that is, the book of dark or mysterious things," which is a pretty good title for a book about algebra."

"I can find pleasures in a book or an artwork or a piece of music that I don't completely understand. In any other serious field of imaginative work there is no necessarily correct interpretation, but in mathematics you must be certain."

"Mathematics is rigid, but for those who comprehend it, the rigidity becomes liberating, a kind of touchstone from which you can launch journeys and to which you can confidently return. Math is modern and historical at the same time."

"Pi has to do with anything that involves periodicity or cyclical behavior."

"I have heard: mathematics is the craft of creating new knowledge from old using deductive logic and abstraction. The theory of formal patterns. Mathematics is the study of quantity."

"Darwin said, "A mathematician is a blind man in a dark room looking for a black cat which isn't there."

"No scripture is as old as mathematics is. All the other sciences are younger, most by thousands of years. More than history, mathematics is the record that humanity is keeping of itself. History is subjective and can be revised or manipulated or erased or lost. Mathematics is objective and permanent."

"$A^2 + B^2 = C^2$ was true before Pythagoras had his name attached to it, and will be true when the sun goes out and no one is left to think of it. It is true for any alien life that might think of it, and true whether they think of it or not. It cannot be changed. So long as there is a world with a horizontal and a vertical, a sky and a horizon, it is inviolable and as true as anything that can be thought."

"Because of mathematics' insistence on proof, it can tell us, within the range of what it knows, what happens time after time."

"The belief that mathematics exists somewhere else than within us, that it is discovered more than created, is called Platonism, after Plato's belief in a non-spatiotemporal realm that was the region of the perfect forms of which the objects on earth were imperfect reproductions."

"In Book 7 of Republic, Plato has Socrates say that mathematicians are people who dream they are awake."

"It led me to realize that while I had learned a lot of things in my grown-up life, I had spent very little time trying to learn something that was difficult for me to learn."

"There is a degree of uneasiness in learning a subject that taxes one's capacities. There is additional uneasiness if one lost to it on the first encounter. I worry not only, Am I up to it, but, I wasn't up to it before, will I be up to it now? And how much of having not been up to it was from how I was taught and how much rests with me and my ableness to learn?"

"Learning is a form of adaptation and of receptivity. Learning math more complicated than arithmetic means absorbing and remembering a wagonload of information and then using it to reason."

"The ability to learn mathematics is thought to decline around forty, when the brain begins slowing its handling of procedural operations such as calculating."

"No idea is bad unless a person is uncritical. Accepting a guess as a truth, as superstitious people do, is misguided, but so is ignoring a guess, as pedantic people do. As regards ideas, it is only bad not to have any."

"A mathematical solution is absolute, although in How Not to Be Wrong, Jordan Ellenberg points out that a lifetime might not be sufficient to achieve it. "Fermat's problem took 350 years," he writes. (Fermat's problem, usually called Fermat's last theorem: no three positive integers A, B, and C satisfy the equation $A^n + B^n = C^n$ if the value for n is a whole number greater than 2.)"

"Mathematics has two pursuits: to find patterns and to prove that the patterns are lasting ones."

"Learning algebra, a person is crossing territory on which footprints have been left since antiquity. Some of the trails have been made by distinguished figures, but the bulk of them have been left by ordinary people such as me."

"Numbers are similar to language in that no one would say that there weren't thoughts before language, although language appears to make possible a means of naming and ordering thoughts and of having complex thoughts that wasn't possible before language."

"Numbers appear to have been found more than created."

"Numbers have a kind of sway over us, but we have no sway over them. Nothing we do can change what they are or how they behave. They were here before us and will remain if we disappear. They are always past, present, and future, and we cannot be sure that we are."

"All objects other than numbers live either in the world of space and time or in the realm of the imagination, which is a precinct of space and time. By the nature of their dual identity, numbers can travel between these places and connect them."

"Numbers are a mystery enfolded into ordinary life. They surround us the way radio waves and dark matter do, is how I think of it, and, like hurricanes and white sharks and big cats, they suggest the edges of the inapprehensible."

"Numbers did not initially provoke wonder or reverence. They did that later, partly because they invoked notions of infinity and therefore of God, and, after that, because they appeared to be a language in which nature could be expressed."

"Leibniz somewhere remarks that music is the pleasure the human mind receives from counting without knowing it is counting."

"It may be beguiling to know that there is no last number, but it is an observation more about infinity than about the hidden qualities of numbers. It might provoke awe, but it is also a mere fact involving the simplest mathematical proof there is, a child can come up with it on his or her own and often does: for any last number n, there is n + 1."

"Kepler exemplifies the premodern belief that practical mysteries conceal the divine. According to Pauli, he thought that the theorems of geometry "have been in the spirit of God since eternity," and that God was represented by the sphere. "The Father is the center," Kepler wrote. "The Son is in the outer surface, and the Holy Ghost is in the equality of the relation between point and circumferences"

"Hegel somewhere remarks that the reason there are so many examples of childhood prodigies in mathematics is that mathematics doesn't involve (as music doesn't either) a grasp of the complexities of mature life, which need experience to make sense of them."

"A Platonist believes that the native territory of numbers and mathematics is somewhere else than in physical reality. This elsewhere is abstract, indifferent to us, was here before us, and will be after."

"About 65 percent of mathematicians are Platonists, at least that is the figure I find most often. The bulk of them tend to endorse a type of passive Platonism that allows that mathematics has aspects that human minds cannot account for, its capaciousness, for one thing, as well as objects that are inherently too large and complex for human minds to encompass."

"For the mathematician and physicist Roger Penrose, to say that a mathematical statement is Platonic is to assert that it is true objectively—true, that is, whether anyone believes it or not."

"Formalism is the brainiest. It says that mathematics is simply a pastime, a game played by rules and properties that have meaning only within the game."

"Intuitionism also believes that mathematics is the mind's creation, and that it doesn't describe anything realistic."

"Endorsing Platonism means allowing the possibility of the world's being more capacious than we can conceive. Evolution might be progressing on a grand scale, too. All that we are meant to know or might be capable of knowing could be unfolding over a longer span than a human life or even generations of lives can enclose. We might not yet be equipped to think all that there is to be thought."

"Mathematics, which settles all questions it is capable of settling, and has plenty of pronouncements about real and imaginary worlds, offers very little about where it comes from."

"With mathematics it is impossible to avoid metaphysical questions. It begins, after all, with an unanswerable question: Where do numbers come from? This is one reason mathematics is bound up with spiritual explanations."

"Mathematicians use the word beauty without irony or hesitation and differently from the way it is used in aesthetics. For mathematicians beauty is more a quality than an appraisal."

"For mathematicians notions of mathematical beauty accord with a belief that the laws of the natural world, which mathematics frequently articulates, and the relations of whatever objects or circumstances mathematics is describing have a harmony, whether in the way that water moves or the orbits of the planets or the operations of shapes too complicated to see. Beauty in mathematical explanations is often a reflection of something glimpsed of a structure that seems to fit the world, although there is no explanation for why it should."

"All the mathematicians I knew told me that I would like geometry. Algebra was necessary and tolerable, but geometry had pleasures."

"Plato said, 'Geometry is knowledge of what always is. It draws the soul towards truth, and produces philosophical thought.'"

"Mathematics being the same everywhere means that one does not have to study Arabic math or Chinese math or Canadian math. Math isn't balkanized."

"There are two kinds of calculus, differential and integral. Differential calculus divides up time—how fast an object has gone far (or how fast a process is accumulating). Integral divides up distance—how far an object has gone fast (or how much is accumulating)."

"Physics is descriptive and not generally concerned with logic. A physicist cares about calculations that describe and predict observed phenomena. There are no external phenomena in mathematics. There are only assertions and axioms and their consequences."

"Einstein said of calculus that it was "the greatest advance in thought that a single individual was ever privileged to make."

"The point of an education is to be introduced to books and art and matters of science and history and thought sufficient to engage one for the rest of one's life or to suggest a means of doing so. An education is a template for knowing where to look for the means to continue one's learning."

"Thinking is a strange pursuit. You are making an approach on an idea. It doesn't want to stay where you can see it clearly or reveal all of itself to you at once. It doesn't hold a shape, it dissolves, it shows you different sides, it slips away. As hard as I try, before long I realize I am thinking of something else, and I have to start over and reconsider and reevaluate, only sometimes advancing."

“Ever tried. Ever failed. No matter. Try again. Fail again. Fail better.” —Samuel Beckett, Worstward Ho!

“Anne Carson’s poem “Essay on What I Think About Most,” which begins, Error. And its emotions. On the brink of error is a condition of fear In the midst of error is a state of folly and defeat. Realizing you’ve made an error brings shame and remorse Or does it?”

“Mathematicians have observed that unlike other arts, mathematics is the result of centuries of shared effort and in that way resembles a church as much as a creative discipline.”

“The Bible is, among many other things of course, a catalog of antique miracles and a plan for moral behavior.”

“A good portion of it involves the attempt to find form for the magical. It can make you feel in the presence of holy events and holy people. It can reduce you to a state of wonder. A not inconsiderable number of people believe that the key to it is mathematics. I regard it as a work of art, a repository of human knowledge. A description of the terrible struggle to remain completely alive in the face of harsh circumstances.”

“Too much faith makes a person liable to being credulous, whereas too much intelligence might restrict faith. Faith that is humility, a belief that we don’t know and might never know the larger design, the intention, the plan if there is one, seems sensible to me, but so does the belief that there is no plan.”

“It does not ask much of the imagination to regard mathematics, especially as it grows more abstruse, as a trail that followed to its end might bring one into the presence of God in the form of a super-axiom.”

“In the eighteenth century, the belief that studying mathematics was a form of worship was exemplified by Maria Gaetana Agnesi, an Italian woman who wrote one of the first significant textbooks on calculus.”

“Plato’s conception of mathematics was metaphysical more than theological. He believed that mathematical truths made the mind receptive to higher truths and an awareness of the Good, an entity different from the Greek gods in that it didn’t look like a person or have supernatural attributes.”

“I did not foresee that learning adolescent math would lead me to notions of divinity. In my defense I will point out that I did not blaze a trail; I followed footprints worn into history.”

“Why have I found this subtext of God knowing so interesting? Partly because it was unexpected, the notion that numbers are hidden in the world and the divine might be hidden in numbers, but also because, like many people, I want to believe in something more than the ordinary terms of life.”

“Studying mathematics made me aware of a natural structure, elusively apparent and perhaps ultimately impenetrable. An implicit orderliness. An unfolding, moment by moment, on an apparently spectacular scale of something that no force can interrupt, something that is perhaps force itself. A trembling quality to life, both fearsome and fragile, a pattern that even to a novice like me is as clear as the grain in a piece of wood.”